REPLY Earth Planets Space, 52, 651–653, 2000

Reply to "Comment on 'Time evolution of the fluid flow at the top of the core. Geomagnetic jerks' by M. Le Huy, M. Mandea, J.-L. Le Mouël, and A. Pais"

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(Received September 13, 2000; Accepted September 18, 2000)

Andrew Jackson's comment is sensible and draws attention to the fact that the lack of magnetic field intensity data prior to 1832 introduces some non-uniqueness in the computed fluid flow at the core-mantle boundary (CMB) in the frozen flux approximation. The problem of modeling the field from directional values is not trivial. Proctor and Gubbins (1990) showed that in some situations even the morphology may not be recovered if one lacks the intensity measurements. However, the only uncertainty experienced by the *ufm2* model (Bloxam and Jackson, 1992; thereafter B& J) is on the amplitude of the geomagnetic field, and not on its morphology because of the mainly dipolar structure of the field (Hulot *et al.*, 1997).

The question raised is interesting; it could indeed be expected that the detailed time evolution of the fluid flow at the CMB be possibly affected by the uncertainty on $\alpha(t)$. But, in fact, we reckon that the most stable features of the flow which are of interest to us (in Le Huy *et al.*, 2000) would not be significantly affected. In order to show it, let us start from Jackson's equation:

$$\frac{\partial B_r^{ufm}}{\partial t} + B_r^{ufm}\frac{\partial \ln\alpha}{\partial t} = -\nabla_h \cdot \left(\boldsymbol{u} B_r^{ufm}\right) \tag{1}$$

which can also be written in the following way

$$\frac{\partial B_r'}{\partial t} = -\nabla_h \cdot \left(\boldsymbol{u} B_r^{ufm}\right), \tag{2}$$

where $\frac{\partial B_r'}{\partial t}$ is an effective secular variation term. This implies that the secular variation coefficients to be used in the inversion should be

$$\left(\dot{g}_n^m\right)^{ufm} + \left(g_n^m\right)^{ufm}\frac{\partial \ln\alpha}{\partial t} \tag{3}$$

instead of $\left(\dot{g}_n^m\right)^{ufm}$ alone as we did (the very same reasoning applies for the h_n^m coefficients).

The problem now is to estimate the term that we have neglected. To this end, we considered the 1840–1990 epoch for which the B_r^{ufm1} is known and corresponds to the true model (the B & J *ufm1* model was determined using magnetic field intensity data) and computed for this epoch a fictive model B_r^{ufm3} in the following way

$$\left(g_n^m\right)^{ufm3} = \frac{\left(g_n^m\right)^{ufm1}}{\alpha(t)}; \qquad \left(h_n^m\right)^{ufm3} = \frac{\left(h_n^m\right)^{ufm1}}{\alpha(t)} \tag{4}$$

with

$$\left(g_1^0\right)^{ufm3} = \kappa\,(t-1840) + \left(g_1^0\right)^{ufm1}(1840) \tag{5}$$

where κ is a constant, and:

$$\begin{aligned}\left(g_1^0\right)^{ufm3}(1840) &= \left(g_1^0\right)^{ufm1}(1840)\\ &= \left(g_1^0\right)^{ufm2}(1840)\end{aligned} \tag{6}$$

so that the $\left(g_1^0\right)^{ufm3}$ and $\left(g_1^0\right)^{ufm1}$ series for both periods 1690–1840 and 1840–1990 are continuous at 1840 as shown by Fig. 1.

We computed κ in (5) by searching the best least squares linear fit to $\left(g_1^0\right)^{ufm1}$, and obtained $\kappa = 15.5$ nT/yr, a value very similar to the value used by B & J for 1690–1840. Function $\alpha(t)$ is then easily computed through

$$\alpha(t) = \frac{\left(g_1^0\right)^{ufm1}(t)}{\left(g_1^0\right)^{ufm3}(t)}, \tag{7}$$

and is shown in Fig. 2. We then computed $\left(g_n^m\right)^{ufm3}$ and $\left(h_n^m\right)^{ufm3}$ from (4) for other values of m and n and compared, for some of the most important field coefficients, $\left(g_n^m\right)^{ufm3}$ with $\left(\dot{g}_n^m\right)^{ufm3} + \left(g_n^m\right)^{ufm3}\frac{\partial \ln\alpha}{\partial t}$ (and the same for h_n^m) (see Fig. 4).

From Figs. 3 and 4 we can see that, for this simulated situation, the effect of the term depending on $\alpha(t)$ does modify significantly $\dot{g}_1^0$ (by its order of magnitude), but is inferior to or of the order of magnitude of the uncertainties associated to secular variation models for the other $\dot{g}_n^m$ and $\dot{h}_n^m$ coefficients. The error that is being committed in taking $g_1^0(t)$ linear in time is in fact very small compared to the order of magnitude of g_1^0 itself (see Fig. 1), with the result that $\alpha(t)$ is very nearly unity (see Fig. 2) and $\ln\alpha(t)$ is very nearly zero for all the 1840–1990 time span. As to $\frac{\partial \ln\alpha}{\partial t}$, it has very small

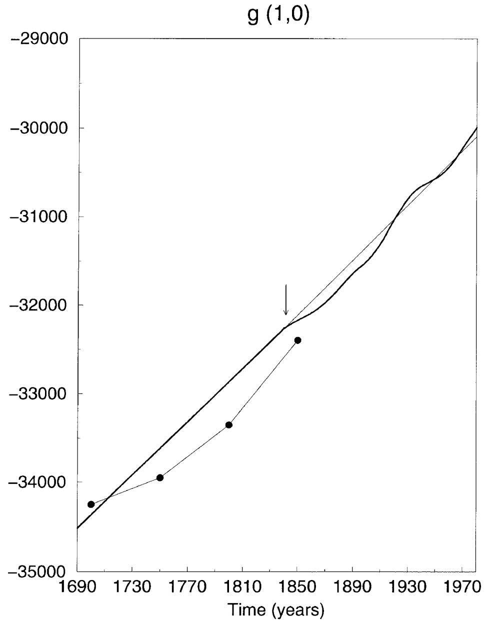

Fig. 1. The g_1^0 coefficient for the whole considered period from the models *ufm1* and *ufm2* (bold solid line). After 1840 the linear regression to *ufm1* is also indicated (thin solid line). The values of the BKC's model are indicated by full circles.

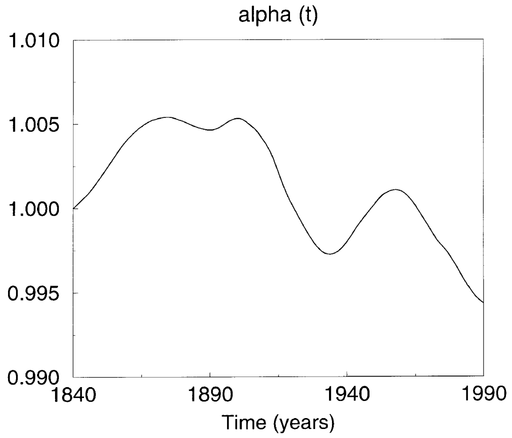

Fig. 2. Function $\alpha(t)$ simulated for the 1840–1990 period.

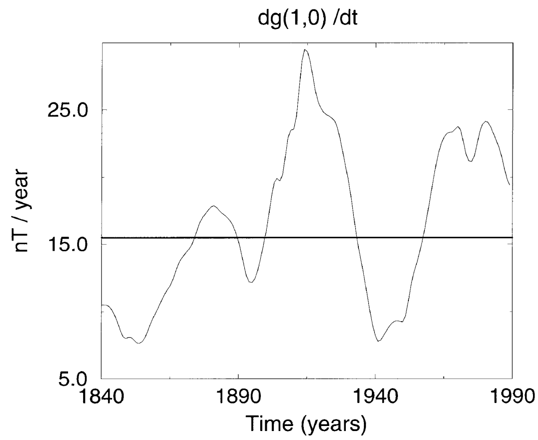

Fig. 3. Rate of change of $\dot{g}_1^0(t)$ for the period 1840–1990 (bold solid line: the values coresponding to the *ufm3* fictive model; thin solid line: values corrected by the term $\left(g_1^0\right)^{ufm3} \frac{\partial \ln \alpha}{\partial t}$).

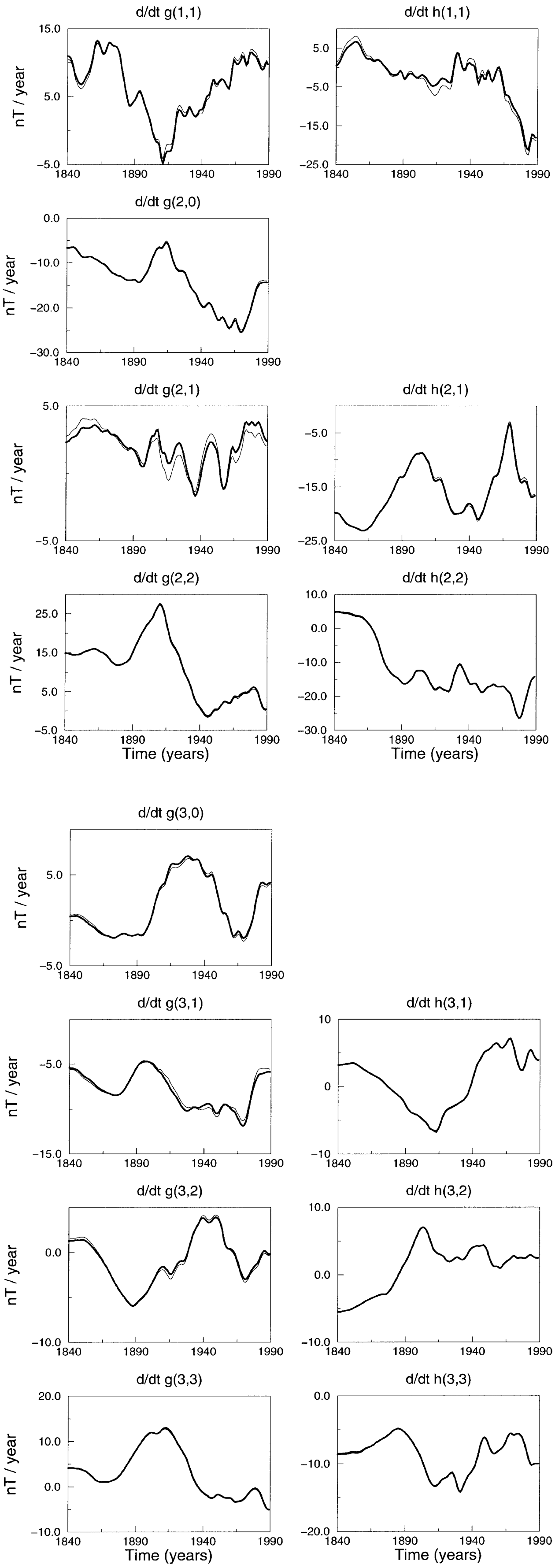

Fig. 4. Rate of change of $g_n^m(t)$, $h_n^m(t)$ with $n, m \leq 3$ for the period 1840–1990 (bold solid line: the values corresponding to the *ufm3* fictive model; thin solid line: values corrected by the term $\left(g_n^m\right)^{ufm3} \frac{\partial \ln \alpha}{\partial t}$ (and the same for h_n^m)).

values of the order of 10^{-4}. So from (3) and (4) we realize that the secular variation corrective term is significant only for the highest coefficient, g_1^0, which is itself of the order of 10^4. Since the intensity of the main field coefficients drops quickly to values of the order of 10^3 or less for $n > 1$ (or even for $n = 1$ and $m = 1$), the corrective term drops quickly to values of the order of 10^{-1} or less. Supposing that the main field has kept these intrinsic characteristics (namely a nearly linear dependence on time of g_1^0 and the global behavior of the field spectrum) for the last 300 years, we are entitled to extend these conclusions to the 1690–1840 epoch.

So finally, the only doubt raised by Jackson in his comment concerns the importance of $\dot{g}_1^0(t)$ for the conclusions that have been drawn in our article, namely the characteristics of the most stable component of the flow. However, we have carried out the same calculations using the Benkova *et al.* (1974) (BKC) model and obtained the same long time scale features of the flow. Since $g_1^0(t)$ in BKC's model is significantly different from B & J's (see Fig. 1), leading to large differences in $\dot{g}_1^0(t)$, we are confident that our main results are not significantly affected by the indeterminacy pointed out by Jackson.

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